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(Abstract)

Hydraulic fracturing is a water intensive process. Water inputs for fracking are a function of the geology, the amount of recoverable resource such as oil or gas, the number and length of horizontal wellbores along with other factors. It is estimated that approximately 2 to 4 million gallons of water per well are required for wells in the Marcellus Shale region. The estimated amount of water per well in Barnett Shale region in Texas and Oklahoma is about 5 million gallons per well. The amount of groundwater used for fracking in the humid eastern part of the United States is said to be trivial. But fracking in arid and semi-arid regions uses a significant amount of groundwater.

The implementation of hydraulic fracturing is highly controversial and communities where fracking takes place are frequently much divided. The proponents of fracking seem to be people who have ties to the energy industry and those who derive royalty payments from fracking. This group emphasizes the economic benefit of more extensively accessible hydrocarbons and the creation of jobs. The opponents argue that there are serious environmental impacts associated with fracking and include the risk of contaminated groundwater, depleting freshwater, degrading air quality, noise pollution, and the consequential hazards to public health.

A review of existing studies suggests that little attention has been directed to the use of groundwater in fracking and how it may impact other uses of groundwater. The objective of this paper is to contribute to this void and is theoretical in nature. The theory of optimal regional development based on treating an aquifer as an exhaustible resource is extended to include hydraulic fracturing activities. The theoretical model will consider nonenergy use of groundwater as well as energy resource production derived from fracking. The nonenergy use represented by agricultural activity which relies on irrigation. An energy commodity is assumed to be produced by a set of energy firms. The production of this energy commodity is assumed to be a function of water inputs drawn from the aquifer as well as an energy resource that is extracted. A general specification of environmental damages from fracking will be included. A set of socially optimal decision rules is derived and analyzed from the perspective of developing policy guidelines for the use of groundwater resources for activities such as fracking. Next, it is assumed that markets for buying and selling energy resource and groundwater property rights exist. Models for an individual energy firm and farm developed. The marginal decision rules for these economic agents are derived and compared with the joint optimization model for purpose of suggesting potential economic policies.

FRACKING AND REGIONAL ECONOMIC DEVELOPMENT BASED ON AN EXHUASTIBLE RESOURCE: AN ECONOMIC MODEL OF THE TRADEOFFS

Introduction

Hydraulic fracturing or fracking is a well-stimulation technique where rock is fractured by using a hydraulically pressurized liquid consisting of water, sand and chemicals. Fluid consisting of chemicals and sand suspended in water is subjected to high pressure as it is injected into a wellbore to create cracks in deep-rock formations through which natural gas, petroleum, and brine can flow more freely. Small grains of hydraulic fracturing proppants (sand or aluminum oxide) hold the fractures open after the hydraulic pressure is removed from the well. Fracking is usually necessary to gain acceptable flow rates in shale gas, tight gas, tight oil, and coal seam gas wells.

Hydraulic fracturing has generated a great deal of controversy. Proponents of fracking argue that the economic benefits include job creation in locations where fracking takes place as well as more extensively accessible hydrocarbons. The opponents of fracking argue that there are extensive environmental impacts that include the risks of contaminating groundwater, depleting fresh water, degrading air quality, potentially triggering earthquakes, noise pollution, surface pollution and possible hazards to public health and the environment. Burnett (2015) provides an overview of a set of recent papers that address some of the issues related to fracking.

Hydraulic fracturing is a water intensive process (Muehlenbachs and Olmstead, 2014). Water inputs for fracking are a function of the geology, the amount of recoverable resource such as oil or gas, the number and length of horizontal wellbores along with other factors. It is estimated that approximately 2 to 4 million gallons of water per well are required for wells in the Marcellus Shale region. The estimated amount of water per well in the Barnett Shale region in

Texas and Oklahoma is about 5 million gallons per well. The amount of groundwater used for fracking in the humid eastern part of the United States is said to be trivial. But fracking in arid and semi-arid regions uses a significant amount of groundwater. Moreover, the natural resource issues that arise with fracking tend to be localized in nature.

A review of existing studies suggests that little attention has been given to the use of groundwater in fracking and how it may impact other uses of groundwater. The objective of this paper is to contribute to this void and is theoretical in nature. The theory of optimal regional development based on treating an aquifer as an exhaustible resource is extended to include hydraulic fracturing activities. The theoretical model will consider nonenergy use of groundwater as well as energy resource production derived from fracking. The nonenergy use is represented by agricultural activity which relies on irrigation. An energy commodity is assumed to be produced by a set of energy firms. The production of this energy commodity is assumed to be a function of water inputs drawn from an aquifer as well as an energy resource that is extracted. A general specification of environmental damages from fracking will be included. A set of socially optimal decision rules is derived and analyzed from the perspective of developing policy guidelines for the use of groundwater resources for activities such as fracking. Next, it is assumed that markets for buying and selling energy resource and groundwater property rights exist. Models for an individual energy firm and farm developed. The marginal decision rules for these economic agents are derived and compared with the joint optimization model for purpose of suggesting potential economic policies.

The contributions of this paper are the following. First, an analytical model showing the competing demands for water withdrawals between fracking activity for producing an energy commodity such as natural gas or oil and irrigation for agriculture is formally developed. This model includes explicit representation of the potential damages from fracking where the damage function formulation includes both the generation of contributions to the pollution stock each time period from fracking activity and the existing stock of pollution as arguments. The second contribution of this paper is a set of marginal decision rules which explicitly capture the nature of marginal social opportunity costs that must be accounted for in order for socially optimal time paths for the use of the exhaustible energy resource and aquifer to evolve. The third contribution is a set of model structures for a set of individual energy firms and farms assuming the existence of markets for trading energy resource and groundwater property rights. Two key results emerge here. First, the appropriate marginal user costs of an exhaustible groundwater resource are fully internalized by the property right market price charged for water used in both fracking and irrigation activities. Second, the value of damages related to fracking are not internalized in the extraction of groundwater for either use. This implies that the extraction rate for groundwater in a market where groundwater rights are traded is likely to exceed the socially optimal rate of extraction of groundwater. It is also shown that the marginal damages for extraction of the exhaustible energy resource are not accounted for in the market for trading rights for the energy resource. It can also be included that rate of extraction for the energy resource that evolves from the property rights market will exceed the socially optimal rate of resource extraction.

Joint Resource Maximization Model

The joint resource maximization model used in this research is developed in this section. The work by Howe (1987) on optimal regional development which is based on an aquifer classified as an exhaustible resource is the starting point for this research. Other relevant modeling formulations include Burt and Cummings (1970), Bohi and Toman (1984), Cummings et al. (1975), Pakravan (1981, 1984) and Willett and Sharda (1988). Two types of competing water-intensive activities are represented. In particular, a water-intensive form of energy production is assumed to compete with agricultural activity and the sole source of water supply for these two activities is an exhaustible aquifer. The index i $(i = 1, \dots, I)$ is used to denote energy firms and the index j $(j = 1, \dots, J)$ is used to represent firms engaged in agricultural activity.

Consider the production of an "energy" commodity such as oil or natural gas with hydraulic fracturing or fracking. It is assumed that there exists a production function relationship for an energy commodity based on the use of a "basic" exhaustible energy resource (such as shale) and water. Let E_{it} denote the amount of energy commodity produced by firm i in period t, R_{it} the amount of energy resource used by firm i in period t, and Z_{it} the amount of water used by firm i to produce the energy commodity in period t. The energy commodity production function is stated as

$$E_{it} = f_{it}(R_{it}, Z_{it}) \tag{1}$$

The properties for this production function are assumed to be the following: $\frac{\partial f_{it}}{\partial R_{it}} > 0$; $\frac{\partial^2 f_{it}}{\partial R_{it}^2} < 0$, $\frac{\partial f_{it}}{\partial Z_{it}} > 0$, $\frac{\partial^2 f_{it}}{\partial f_{it}^2} < 0$, and $\frac{\partial^2 f_{it}}{\partial R_{it} \partial Z_{it}} \neq 0$.

Next consider the cost function for producing the energy commodity. Let N_t represent the total amount of the energy resource remaining in the ground in period t. The cost function for firm i is represented as $C_{it}(R_{it}, N_t)$ and the properties of this cost function are assumed to be the following: $\frac{\partial C_{it}}{\partial R_{it}} > 0$, $\frac{\partial^2 C_{it}}{\partial R_{it}^2} < 0$, $\frac{\partial C_{it}}{\partial N_t} < 0$, $\frac{\partial^2}{\partial N_t^2} > 0$, and $\frac{\partial^2 C_{it}}{\partial R_{it} \partial N_t^2} < 0$. Let W_t represent the stock of water remaining in the ground in period t. The cost of using water in the production of the energy commodity is denoted as $G_{it}(Z_{it}, W_t)$ and the properties of this cost function are assumed to be the following: $\frac{\partial G_{it}}{\partial Z_{it}} > 0$, $\frac{\partial^2 G_{it}}{\partial Z_{it}^2} < 0$, $\frac{\partial G_{it}}{\partial W_t} < 0$, $\frac{\partial^2 G_{it}}{\partial W_t^2} > 0$, and $\frac{\partial^2 G_{it}}{\partial Z_{it} \partial W_t} < 0$.

Let the gross benefit function for firm i in the production of the energy commodity in period t be denoted as $B_{it}(E_{it})$. The energy firm's profit function in period t is given as follows:

$$\Pi_{it}(E_{it}) = B_{it}(E_{it}) - C_{it}(R_{it}, N_t) - G_{it}(Z_{it}, W_t) \quad (2)$$

The production function given by equation (1) represents the production of the energy commodity.

The second type of production activity is agriculture. It is assumed that the primary input for agricultural production is water. Let x_{jt} represent the amount of water pumped by farm j in period t and Q_{jt} represent the quantity of output produced by farm j in period t. Agricultural production is represented by the following production function:

$$Q_{jt} = g_{jt}(x_{jt}) \quad (3)$$

This function is assumed to have the following properties: $\frac{\partial g_{jt}}{\partial x_{jt}} > 0$, and $\frac{\partial^2 g_{jt}}{\partial x_{jt}^2} < 0$. The cost function for agricultural activities is stated as $F_{jt}(x_{jt}, W_t)$ and is assumed to have the following properties: $\frac{\partial F_{jt}}{\partial x_{jt}} > 0$, $\frac{\partial^2 F_{jt}}{\partial x_{jt}^2} < 0$, and $\frac{\partial^2 F_{jt}}{\partial x_{jt} \partial W_t} < 0$.

Let the agricultural firm's gross benefit function be stated as $B_{jt}(Q_{jt})$ with $\frac{\partial B_{jt}}{\partial Q_{jt}} > 0$ and $\frac{\partial^2 B_{jt}}{\partial Q_{jt}^2} < 0$. The profit function for farm j in period t is stated as

$$\Pi_{jt}(Q_{jt}) = B_{jt}(Q_{jt}) - F_{jt}(x_{jt}, W_t). \quad (4)$$

The production function for agricultural output is given by equation (3).

The next component of the overall model is the pollution stock and the corresponding damage function. Key papers here include Lyon and Lee (2004), Farzin (1996), and Willett and Sharda (1988). The treatment of the pollution stock and damage function follows the modeling strategies shown in Farzin (1996) most closely. Important elements of the specifications include environmental damage as a function of both the flow of environmental pollution or emissions as and the stock of pollution. Farzin (1996) also includes an "emission" abatement technology. Let Y_t represent the pollution stock in period t that is related to the production of the energy commodity. Also let y_{it} represent the amount of pollution generated in period t by firm i from its production of the energy commodity. It is assumed that the generation of pollution is represented by the following relationship:

$$y_{it} = k_{it}(E_{it}) \tag{5}$$

where $\frac{\partial k_{it}}{\partial E_{it}} > 0$. It is also assumed that each firm i undertakes some sort of abatement or remediation activity n_{it} in each period and the cost of this activity is represented as $e_{it}(n_{it})$. The properties of this cost function are assumed to be the following: $\frac{\partial e_{it}}{\partial n_{it}} > 0$ and $\frac{\partial^2 e_{it}}{\partial n_{it}^2} > 0$. This cost function must be incorporated into the energy firm's profit function equation (2).

The last functional relationship to be developed is the (aggregate) damage function. The damage function specification used by Farzin (1996) is adopted here. Let the total pollution generated by energy commodity production in each time period be denoted as y_t with

$$y_t = \sum_{i=1}^{I} y_{it}. \tag{6}$$

The damage function is stated as $D_t(y_t, Y_t)$ and is assumed to have the following properties: $\frac{\partial D_t}{\partial y_t} > 0$, $\frac{\partial^2 D_t}{\partial y_t^2} < 0$, and $\frac{\partial D_t}{\partial Y_t} > 0$. The damage function is assumed to be separable in its arguments.

The functions developed in the previous paragraphs can be used to form the dynamic joint resource optimization model. Let $\beta^t = (1 + r)^{-t}$ represent a discounting factor with r being an appropriately determined interest rate. The optimization model is as follows:

$$Max \sum_{t=0}^{T-1} \beta^t \left\{ \sum_{i=1}^{I} [B_{it}(E_{it}) - C_{it}(R_{it}, N_t) - G_{it}(Z_{it}, W_t) - e_{it}(n_{it})] + \sum_{j=1}^{J} [B_{jt}(Q_{jt}) - F_{jt}(x_{jt}, W_t)] - D_t(y_t, Y_t) \right\} + \beta^t H_T(N_T) + \beta^T V_T(W_t) - \beta^{T-t} M_T(Y_T) \tag{7}$$

Subject to

$$E_{it} = f_{it}(R_{it}, Z_{it}) \;\; (\mu_{it}) \tag{8}$$

$$(i = 1, \dots, I)$$

$$(t = 0, \dots, T-1)$$

$$Q_{jt} = g_{jt}(x_{jt}) \;\; (\varepsilon_{jt}) \tag{9}$$

$$(j = 1, \dots, J)$$

$$(t = 0, \dots, T-1)$$

$$y_t = \sum_{i=1}^{I} y_{it} \;\; (\theta_t) \tag{10}$$

$$(t = 0, \dots, T-1)$$

$$y_{it} = k_{it}(E_{it}) \;\; (\alpha_{it}) \tag{11}$$

$$(i = 1, \dots, I)$$

$$(t = 0, \dots, T-1)$$

$$N_{t+1} = N_t - \sum_{i=1}^{I} R_{it} \;\; (\lambda_{t+1}) \tag{12}$$

$$(t = 0, \dots, T-1)$$

$$W_{t+1} = W_t - \sum_{i=1}^{I} Z_{it} - \sum_{j=1}^{J} x_{jt} \;\; (\psi_{t+1}) \tag{13}$$

$$(t = 0, \dots, T-1)$$

$$Y_{t+1} = Y_t + y_t - \sum_{i=1}^{I} n_{it} \quad (\phi_{t+1}) \tag{14}$$

$$(t = 0, \ldots, T-1)$$

The variables in parentheses to the right of each equation in the constraint set are Lagrangean multipliers.

The components of the optimization model are as follows. Equation (7) represents the model objective function and consists of the following. The first set of terms in brackets represents the energy firm profits. This is equation (2) with the firm abatement cost function included. The second set of expressions in brackets represents farm profits and the last term is the environmental damage function for each time period. The last two terms in equation (7) are, respectively, terminal value function for the energy resource and the terminal value function for the groundwater stock and the terminal damage function for the pollution stock. These functions will be discussed in more depth in the next section.

The remaining components of the model equations (8)-(14) of course represent the model constraint set. Equations (9)-(11) are functional relationships described previously in this section. Equation (12) is the transition equation for the energy resource stock and equation (13) is the transition equation for the groundwater stock. Finally, equation (14) is the transition equation for the pollution stock.

Stock Variable Shadow Prices

The stock variable shadow prices are evaluated first in this section because they provide significant insight on the future cost of resource use in the current period and the impact of the pollution stock. (The derivations in this section are available from the author upon request.) The evaluation of these shadow prices include explicit use of the terminal value functions which are important with respect to resource stock valuation with a finite decision making horizon.

The first stock resource shadow price is for the exhaustible energy resource and is given

$$\beta\lambda_{t+1} = -\sum_{\tau=t+1}^{T-1}\left[\beta^{\tau-t+1}\sum_{i=1}^{I}\frac{\partial C_{i\tau}}{\partial N_\tau}\right] + \beta^{T-t}\frac{\partial H_T}{\partial N_T} \quad (15)$$

by equation (15). The right had side of this equation is the marginal social opportunity cost of firm i for removing a unit of the energy resource stock in period t. Consider this decision more closely. Lowering the energy resource stock by an incremental unit in period t is expected to increase the cost of removing the energy resource by all energy firms in all remaining periods of the decision-making horizon. In particular, the expression in brackets, the marginal user cost, represents the effect that the decision by energy firm i to remove a unit of the energy resource in period t is expected to have on all future decisions for energy firms also removing the energy resource. Recall that $\frac{\partial C_{i\tau}}{\partial N_\tau}$ is negative, so the marginal user cost for reductions in the energy resource stock are all positive. An alternative interpretation applicable here is that the marginal user cost can be interpreted as the discounted marginal value of the remaining stock of *in situ* energy stock reserves. The effect of this cost is magnified when the length of the decision-making horizon is shortened or when the rate of the interest rate is increased. The last term on the right hand side of equation (15) is the marginal economic value to economic society of the *in situ* energy resource stock for time periods beyond the current decision-making time horizon.

The next stock resource shadow price is for the groundwater stock and is represented by equation (16). For the most part, the interpretations parallel those provided for equation (15) and the

$$\beta\psi_{t+1} = -\sum_{\tau=t+1}^{T-1} \beta^{\tau-t+1}\left(\sum_{i=1}^{I}\frac{\partial G_{i\tau}}{\partial W_\tau} + \sum_{j=1}^{J}\frac{\partial F_{j\tau}}{\partial W_\tau}\right) + \beta^{T-t}\frac{\partial V_T}{\partial W_T} \tag{16}$$

key differences will be highlighted here. The use of groundwater in this modeling exercise includes both energy firms and farmers. The marginal social opportunity cost of an economic agent's decision to pump a unit of groundwater in period t is represented by the expression in parentheses on the right hand side of equation (16) and includes future marginal cost impacts on both energy firms and firms. This marginal user cost has a similar interpretation to that given in equation (15) for the energy resource stock. In particular, the marginal social opportunity cost of an economic agent's decision to pump a unit of groundwater in period t is shown by the terms in brackets. The last term on the right hand side of equation (16) is the marginal economic value to economic society of the in situ groundwater stock for the relevant time period beyond the current decision-making time horizon.

The last stock resource shadow price is for the pollution stock and is given by equation (17).

$$\beta\phi_{t+1} = -\sum_{\tau=t+1}^{T-1}\left[\beta^{\tau-t+1}\frac{\partial D_\tau}{\partial Y_\tau}\right] + \beta^{T-t}\frac{\partial M_T}{\partial Y_T} \tag{17}$$

The shadow price for increasing the pollution stock by one more unit in period t is the discounted sum of all the marginal pollution damages imposed in all future periods of the decision-making horizon and is represented by the expression in brackets. The last term on the right hand side of equation (17) is the marginal damages imposed on economic society by the existing pollution stock in the future periods beyond the current decision-making horizon. This shadow price can also be interpreted as the shadow value of a unit of unpolluted environment. Farzin calls this an environmental scarcity rent.

Marginal Decision Rules and Optimal Resource Use

The rules for making optimal decisions for the use of the energy resource and groundwater resource in the production of the energy commodity are examined first. At time t,

$$\frac{\partial B_{it}}{\partial E_{it}}\frac{\partial f_{it}}{\partial R_{it}} = \frac{\partial C_{it}}{\partial R_{it}} + \beta\lambda_{t+1} + \left[\frac{\partial D_t}{\partial y_t}\frac{\partial k_{it}}{E_{it}}\frac{\partial f_{it}}{\partial R_{it}} - \beta\phi_{t+1}\frac{\partial k_{it}}{\partial E_{it}}\frac{\partial f_{it}}{\partial R_{it}}\right] \tag{18}$$

the optimal time path for using the energy resource to produce the energy commodity is given by equation (18). The left side of this equation represents the marginal benefit of using the energy resource to produce this commodity. The right side of equation (18) represents the marginal social opportunity cost of using the energy resource in this production endeavor. The first component is the marginal opportunity cost of the resources used to extract the energy resource from the ground in period t. The second term is the marginal social opportunity cost incurred when firm i extracts a unit of the energy resource from the ground and is shown in more detail by equation (15). The third term in brackets shows the marginal social damage cost of the use of extracting and using the energy resource in the energy commodity production. This marginal opportunity cost consists of two components. The first component is the increase in marginal damages in period t that arise because of the use the energy resource in producing the energy commodity. The second component is the increase in damages as the stock of pollution increases in period t related to an addition of the energy resource being used. The valuation of for this effect is reflected in equation (17).

The optimal time path for using the groundwater resource to product the energy commodity in period t is given by equation (19). The left side represents the marginal benefits

$$\frac{\partial B_{it}}{\partial E_{it}}\frac{\partial f_{it}}{\partial Z_{it}} = \frac{\partial G_{it}}{\partial Z_{it}} + \beta\psi_{t+1} + \left[\frac{\partial D_t}{\partial y_t}\frac{\partial k_{it}}{\partial E_{it}}\frac{\partial f_{it}}{\partial Z_{it}} - \beta\phi_{t+1}\frac{\partial k_{it}}{\partial E_{it}}\frac{\partial f_{it}}{\partial Z_{it}}\right] \tag{19}$$

using groundwater to produce the energy commodity. The marginal opportunity social costs of the use of groundwater are shown on the right side of this equation. The first term represents the marginal opportunity costs of the resources used in period t to pump groundwater and the second term is the marginal user cost of the groundwater used as discussed with respect to equation (16). The third term represents the impact on marginal damages as more groundwater is used in the current period.

The energy firm's optimal expenditure for abatement or remediation activity is shown by

$$-\beta\phi_{t+1} = \frac{\partial e_{it}}{\partial n_{it}} \tag{20}$$

by equation (20). The expression on the left side of this equation is interpreted as the marginal value of a unit of unpolluted environment in future years resulting from an increase in abatement activities by the energy firm in period t. The right side of equation (20) measures the marginal opportunity cost of an incremental increase in abatement activity in period t by energy firm i.

The last topic to discuss is the optimal path of groundwater pumping by farm j. The time

$$\frac{\partial B_{jt}}{\partial Q_{jt}}\frac{\partial g_{jt}}{\partial x_{jt}} = \frac{\partial F_{jt}}{\partial x_{jt}} + \beta\psi_{t+1} \tag{21}$$

For groundwater pumping in agriculture is given by equation (21). First, the left side of this equation shows the marginal benefit of groundwater use in period t by firm i.

The expression on the right side of equation (21) represents the marginal social opportunity cost of the groundwater use in agriculture and consists of two components. The first term represents the marginal opportunity cost of the resources used by farm j in the current period to pump groundwater. The second term is the marginal user cost of groundwater use in agriculture. This detailed components of this relationship are shown by equation (17).

Individual Firm Decisions with Property Rights Markets

The main objective of the previous sections was to identify a set of socially optimal decision rules that would provide insight for developing a set of policy guidelines for the use of an exhaustible groundwater resource where energy development based on hydraulic fracturing or fracking competes with another type of regional economic development such as agriculture. The energy resource stock is also assumed to be exhaustible and the development of an energy commodity such as oil or gas from shale also includes an accounting of environmental damages.

The particular information used from the marginal decision rules derived from the joint resource maximization problem to design an economic policy, depends upon the particular type of economic institutional structure firms currently operate under. One such example is where there is presumed to be a well-defined property right for the energy resource and also the groundwater resource. In such a case, these property rights are traded in a market setting. The basic idea for an individual agent participating in a market where property rights to a stock resource are well-defined and trades take place through an organized market is based Vernon Smith's (1977) discussion of a "water deed." Different forms of this concept have been developed by Anderson et al. (1983), Fractor (1988), Ghosh and Willett (2012), and Provencher (1988, 1993). An example of this sort of market formulation is developed an analyzed in this section.

Model structures for an individual energy firm and farm assuming property right trading of both the exhaustible energy resource and groundwater are developed in this section. The basic modeling structure shown in Ghosh and Willett (2012) is used here. Let N_{it} represent the number of energy resource rights or permits owned by energy firm i in period t and X_{it} the number of energy resource rights traded by firm i in period t at price p_{Et}. The price of a right is assumed to be constant by the energy firm, being determined in a competitive energy resource permit trading market. In addition, let R_{it} represent the quantity of the energy resource removed by energy i firm in period t.

It is also assumed that the energy firm has an inventory of water rights and buy or sell these rights each time period. Let W_{it} represent the number of groundwater permits held by firm i in period t. Also let the number of groundwater rights bought or sold in period t by firm i be represented by A_{it}. These groundwater permits are assumed to be purchased in a competitive market setting at a market-determined price of p_{Wt}. The energy firm's decision problem is presented as follows:

$$Max \sum_{t=0}^{T-1} \beta^t \{B_{it}(E_{it}) - C_{it}(R_{it}, N_{it}) - G_{it}(Z_{it}, W_{it}) - p_{Et}X_{it} - p_{Wt}A_{it}\} + \beta^T H_{iT}(N_{iT}) + \beta^T V_{iT}\big(W_{it,}\big) \quad (22)$$

Subject to

$$E_{it} = f_{it}(R_{it}, Z_{it}) \;\; (\mu_{it}) \quad (23)$$

$$(t = 0, \;\; , T-1)$$

$$N_{i(t+1)} = N_{it} + X_{it} - R_{it} \;\; \big(\lambda_{i(t+1)}\big) \quad (24)$$

$$(t = 0, \dots, T-1)$$

$$W_{i(t+1)} = W_{it} + A_{it} - Z_{it} \;\; \big(\psi_{i(t+1)}\big) \quad (25)$$

$$(t = 0, \dots, T-1)$$

$$N_{io} = \overline{N}_{i0} \;\; (\lambda_{i0}) \quad (26)$$

$$W_{i0} = \overline{W}_{i0} \;\; (\psi_{i0}) \quad (27)$$

The terms in parentheses following each equation are Lagrangean multipliers.

The equations for the energy firm's decision problem are briefly described. First, equation (22) represents the firm's profit function and includes terminal value functions to show the value of the firm's terminal period holdings of energy resource permit holdings and groundwater permit holdings, respectively. Equation (23) is the energy firm's energy commodity production function and has the same properties as described previously. Equation (24) is the energy firm's state equation for energy resource permits and equation (25) is the firm's state equation for groundwater permit holdings. Equations (26) and (27) show initial holdings of energy resource permits and groundwater permits, respectively, by the energy firm.

Now consider the resource energy stock shadow price which is represented by equation

$$\beta\lambda_{i(t+1)} = -\sum_{\tau=t+1}^{T-1}\left[\beta^{\tau-t+1}\frac{\partial C_{i\tau}}{\partial N_{i\tau}}\right] + \beta^{T-t}\frac{\partial H_{iT}}{\partial N_{it}} \tag{28}$$

(28). The first term on the right side of this equation in brackets shows the energy firm's marginal user cost of extracting one unit of the energy resource in period t. The second term on the right side of the equation represents the impact that extraction in period t will have on the economic value of the energy firm's energy resource permit holdings in the terminal period. It is important to observe at this juncture that equation (28) only reflects opportunity costs that accrue to energy firm i only as a result of its own decisions.

The groundwater stock shadow price is represented by equation (29). The interpretations

$$\beta\psi_{i(t+1)} = -\sum_{\tau=t+1}^{T-1}\left[\beta^{\tau-t+1}\frac{\partial G_{i\tau}}{\partial W_{i\tau}}\right] + \beta^{T-t}\frac{\partial V_{iT}}{\partial W_{iT}} \tag{29}$$

for this equation are similar to those in equation with reference to the groundwater resource and will not be repeated in detail here. This equation represents the energy firm's marginal user cost for its groundwater permit holdings. Once again, these costs are shown to accrue only to energy firm i.

The energy firm's optimal time path for extraction of the energy resource is given by

$$\frac{\partial B_{it}}{\partial E_{it}}\frac{\partial f_{it}}{\partial R_{it}} = \frac{\partial C_{it}}{\partial R_{it}} + \beta\lambda_{i(t+1)} \tag{30}$$

equation (30). The left side of the equation show the marginal benefit energy firm i earns each period t. The first term on the right side of this equation shows the marginal cost to energy firm i of the resources used to extract a unit of the energy in period t while the second term is the energy firm's marginal user cost of extracting a unit of the energy resource in period t.

The energy firm's optimal time path for pumping groundwater is represented by equation

$$\frac{\partial B_{it}}{\partial E_{it}}\frac{\partial f_{it}}{\partial Z_{it}} = \frac{\partial G_{it}}{\partial Z_{it}} + \beta\psi_{i(t+1)} \tag{31}$$

(31). The left side of this equation represents the marginal return to the energy firm from the extraction and use of groundwater in the production of the energy commodity. The first term on the right side of the equation shows the marginal opportunity cost of the resources used to extract a unit of groundwater in period t. The second term is the energy firm's marginal user cost of pumping groundwater.

The optimal time path for purchasing energy resource permits and groundwater permits are given by equations (32) and (33). In each case, the permit purchases or optimal holds of

$$p_{Et} = \beta\lambda_{i(t+1)} \tag{32}$$

$$p_{wt} = \beta\psi_{i(t+1)} \tag{33}$$

permits requires that the market permit prices be equal to the respective resource stock shadow price.

The groundwater resource management decision problem for farm j is represent by

$$Max \sum_{t=0}^{T-1} \beta^t \left[B_{jt}(Q_{jt}) - F_{jt}(x_{jt}, W_{jt}) - p_{Wt}A_{jt}\right] + \beta^T V_{jT}(W_{jT}) \tag{33}$$

Subject to

$$Q_{jt} = g_{jt}(x_{jt}) \quad (\mu_{jt}) \tag{34}$$

$$(t = 0, \dots, T-1)$$

$$W_{j(t+1)} = W_{jt} + A_{jt} - x_{jt} \quad (\psi_{j(t+1)}) \tag{35}$$

$$(t = 0, \dots, T-1)$$

$$W_{j0} = \bar{W}_{j0} \quad (\psi_{j0}) \tag{36}$$

equations (33)-(36). Again, this model formulation follows the groundwater permit trading model shown in Ghosh and Willett (2012). The variables shown in parentheses to right of each the equations in this model are Lagrangean multipliers. Equation (33) is the farm's objective function and is similar to equation (4) with two basic extensions. The first one is the expression $p_{Wt}A_{jt}$ showing the farm's purchase/sales of groundwater permits in period t. The last term in equation (33) is the farm's terminal value function showing the value of the farm's holdings of groundwater permits in the time period at the end of the farm's decision-making horizon. Equation (34) represents the farm's production function and is the same as equation (4). Equation (35) is the farm's transition equation for groundwater permit holdings and equation (36) is a set of initial conditions for the farm's groundwater permit holdings.

The marginal decision rules are examined once again to gain insight into the optimal time for the use of groundwater by farm j. First, consider the shadow price for the groundwater permit stock holdings which is shown in equation (37). The expression in brackets on the right side of

$$\beta\psi_{j(t+1)} = -\sum_{\tau=t+1}^{T-1}\left[\beta^{\tau-t+1}\frac{\partial F_{j\tau}}{\partial W_{j\tau}}\right] + \beta^{T-t}\frac{\partial V_{jT}}{\partial W_{jT}} \tag{37}$$

this equation shows the farm's marginal user cost of pumping groundwater and the discussions of this term are similar to those from previous paragraphs. Once again, notice that this user cost applies to the groundwater stock farm j can claim ownership to. The last term on the right side of equation (37) shows the marginal value of the groundwater rights owned by firm j for future periods beyond the farm's current decision-making horizon.

The farm's optimal path for pumping groundwater is represented by equation (38). The left side of this equation shows the farm's marginal benefit of pumping groundwater in each time

$$\frac{\partial B_{jt}}{\partial Q_{jt}}\frac{\partial g_{jt}}{\partial x_{jt}} = \frac{\partial F_{jt}}{\partial x_{jt}} + \beta\psi_{j(t+1)} \tag{38}$$

period. The first term on the right side of equation (38) shows the marginal opportunity cost of the resources used in the current period to pump groundwater and the second term is the farm's marginal user cost for pumping groundwater in period t.

The optimal time for the farm's decision to buy/sell groundwater permits is given by

$$p_{Wt} = \beta\psi_{j(t+1)} \tag{39}$$

Equation (39). The left side of this equation is the exogenously determined permit price for groundwater permits and this is set equal to the farm's marginal user cost of groundwater permits.

Another perspective can be presented for the energy firm's to buy or sell permits for the energy resource. For each firm i, it can be shown that

$$\frac{\partial B_{it}}{\partial E_{it}}\frac{\partial f_{it}}{R_{it}} - \frac{\partial C_{it}}{\partial R_{it}} = p_{Et} \tag{40}$$

If the left side of equation (40) is greater than the permit price for the energy resource in any period t, the energy firm will purchase permits. If, on the other hand, the left side of this equation is less than the energy resource permit price, the firm has an incentive to sell permits. The energy firm's energy resource permit holdings are in equilibrium if equation (40) holds as a strict equality.

For each energy firm and each farm, the respective marginal decision rules to buy or sell permits for water is the following:

$$\frac{\partial B_{it}}{\partial E_{it}}\frac{\partial f_{it}}{\partial Z_{it}} - \frac{\partial G_{it}}{\partial Z_{it}} = p_{Wt} \tag{41}$$

$$\frac{\partial B_{jt}}{\partial Q_{jt}}\frac{\partial g_{jt}}{\partial x_{jt}} - \frac{\partial F_{jt}}{\partial x_{jt}} = p_{Wt} \tag{42}$$

A set of incentives for buying and selling groundwater permits implied by equations (41) and (42) parallel the discussions for equation (40) for the energy resource permits and is not repeated here.

The various functions of the permit prices for the energy resource and groundwater resource in each time period can be explored. First, equation (40) for the energy resource and equations (41) and (42) for groundwater make clear the role that the permit prices play in each agent's decision to buy, or sell the respective permits. In equilibrium, the following conditions hold in each period for an agent's holdings of energy resource permits:

$$p_{wt} = -\sum_{\tau=t+1}^{T-1} \beta^{\tau-t+1}\left[\frac{\partial G_{i\tau}}{\partial W_{i\tau}}\right] + \beta^{T-t}\frac{\partial V_{iT}}{\partial W_{iT}} \tag{43}$$

The following conditions hold in equilibrium for groundwater permits for each agent:

$$p_{wt} = -\sum_{\tau=t+1}^{T-1} \left[\beta^{\tau-t+1} \frac{\partial F_{j\tau}}{\partial W_{j\tau}}\right] + \beta^{T-t} \frac{\partial V_{jT}}{\partial W_{jT}} \tag{44}$$

$$p_{Et} = -\sum_{\tau=t+1}^{T-1} \left[\beta^{\tau-t+1} \frac{\partial C_{i\tau}}{\partial N_{i\tau}}\right] + \beta^{T-t} \frac{\partial H_{iT}}{\partial N_{iT}} \tag{45}$$

The right side of equation (43) shows the marginal return to the energy firm when it holds a stock of energy resource permits. The benefit of an additional energy resource permit is the discounted value of the decrease in marginal extraction costs over the remaining portion of the firm's decision horizon when the permit is purchased. Equation (43) is another way to shows the firm's marginal benefit of maintaining its stock of claims to the energy resource stock. This will equal the marginal cost of the permit at the margin. A similar set of discussions apply to equations (44) and (45) for groundwater permits and are not repeated here.

The economic efficiency with respect to the use of the energy resource stock and groundwater stock when transferable permits are bought and sold in a market setting can also be addressed. The economic agents treat the market prices p_{Et} and p_{Wt} as fixed in any time period, and each firm pays the same price for an energy resource permit and the groundwater permit market participants pay the same price for a permit in that market each time period. It can be shown that equilibrium in the energy resource permit market and the groundwater permit market that the marginal user cost each agent incurs in the extraction of a unit of the energy resource is equalized across all energy firms in the energy resource permit market in each period of the decision-making horizon. In a similar way, it can be shown that the marginal user cost each agent incurs in pumping a unit of groundwater is equalized across all agents in each period of the decision-making horizon. Given that each agent in the energy resource market is invested in ownership of well-defined units of the energy resource stock, it can be concluded that the market price of an energy resource permit captures the value of the remaining stock of the energy resource remaining in the ground and each agent is compensated for energy resource conservation decisions. A similar line of reasoning holds for managing the groundwater resource through a permit market where well-defined property rights for claims to the stock of a groundwater stock exist.

The arguments outlined in the previous paragraphs lead to the conclusion that the permit markets based on well-defined claims to resource stocks provide a set of efficient outcomes as institutions for managing the energy resource stock and aquifer. But the environmental damages from hydraulic fracturing remain problematic since they are not accounted for in the individual energy firm's marginal decision rule for extracting the energy resource equation (30) and pumping groundwater, equation (31). The environmental damages related to fracking could be internalized with an economic incentive system such as a tax such as that suggested in Willett and Sharda (1988).

Summary and Conclusions

Hydraulic fracturing or fracking is a well-stimulation technique where rock is fractured by using a hydraulically pressurized liquid consisting of water, sand and chemicals. Proponents of fracking argue that the economics benefits of this activity include job creation in locations where fracking takes place as well as more extensively accessible hydrocarbons. The opponents of fracking argue that there are extensive environmental impacts from fracking. In addition, existing evidence suggests that fracking is a water intensive process. Fracking in arid and semi-arid regions uses a significant amount of groundwater. The natural resource issues that arise with fracking tend to localized in nature.

The use of groundwater in fracking and how it may impact other uses of groundwater does not seem to have examined in much detail. The objective of this paper was to contribute to this void and is theoretical in nature. The theory of optimal regional development based on treating an aquifer as an exhaustible resource is extended to include hydraulic fracturing activities. The theoretical model will consider non-energy use of groundwater as well as energy resource production derived from fracking. The non-energy use is represented by agricultural activity which relies on irrigation. An energy commodity is assumed to be produced by a set of energy firms. The production of this energy commodity is assumed to be a function of water inputs drawn from an aquifer as well as an energy resource that is extracted. A general specification of environmental damages from fracking will be included. A set of socially optimal decision rules is derived and analyzed from the perspective of developing policy guidelines for the use of groundwater resources for activities such as fracking. Next, it is assumed that markets for buying and selling energy resource and groundwater property rights exist. Models for an individual energy firm and farm developed. The marginal decision rules for these economic agents are derived and compared with the joint optimization model for purpose of suggesting potential economic policies.

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